Sheet 3

1. (3.4) Show that the 8-puzzle states are divided into two disjoint sets, such that any state is reachable from any other state in the same set, while no state is reachable from any state in the other set. (*Hint:* See Berlekamp et al. (1982).) Devise a procedure to decide which set a given state is in, and explain why this is useful for generating random states.
2. (3.15) Consider a state space where the start state is number 1 and each state k has two successors: numbers $2k$ and $2k+1$.
   (a) Draw the portion of the state space for states 1 to 15.
   (b) Suppose the goal state is 11. List the order in which nodes will be visited for breadth-first search, depth-limited search (with limit 3), and iterative deepening search.
   (c) How well would bidirectional search work on this problem? What is the branching factor in each direction of the bidirectional[1] search?
   (d) Does the answer to (c) suggest a reformulation of the problem that would allow you to solve the problem of getting from state 1 to a given goal state with almost no search?
   (e) Call the action going from k to $2k$ Left, and the action going to $2k+1$ Right. Can you find an algorithm that outputs the solution to this problem without any search at all?
3. (3.18) Describe a state space in which iterative deepening search performs much worse than depth-first search (for example, $O(n^2)$ vs $O(n)$.
4. (3.20) Consider the vacuum-world problem defined in Figure 2.2.

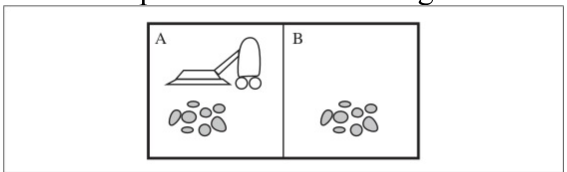

Figure 2.2 A vacuum-cleaner world with just two locations.

   (a) Which of the algorithms defined in this chapter would be appropriate for this problem? Should the algorithm use tree search or graph search?
   (b) Apply your chosen algorithm to compute an optimal sequence of actions for a 3×3 world whose initial state has dirt in the three top squares and the agent in the center.
   (c) Construct a search agent for the vacuum world, and evaluate its performance in a set of 3×3 worlds with probability 0.2 of dirt in each square. Include the search cost as well as path cost in the performance measure, using a reasonable exchange rate.
   (d) Compare your best search agent with a simple randomized reflex agent that sucks if there is dirt and otherwise moves randomly.
   (e) Consider what would happen if the world were enlarged to n×n. How does the performance of the search agent and of the reflex agent vary with n?
5. (3.21) Prove each of the following statements, or give a counterexample:
   (a) Breadth-first search is a special case of uniform-cost search.
   (b) Depth-first search is a special case of best-first tree search.
   ~~(c) Uniform-cost search is a special case of A* search~~

References:

Berlekamp, E. R., Conway, J. H., and Guy, R. K. (1982). Winning Ways, For Your Mathematical Plays. Academic Press.

1 The idea behind bidirectional search is to run two simultaneous searches—one forward from the initial state and the other backward from the goal—hoping that the two searches meet in the middle.